

100% Solar heated house with attic heat store for Ottawa, Ontario

Calculation of heat loss and solar collector area

This is a MathCAD 8 file.

David Delaney, Ottawa, November 21, 2004

The house is a bungalow (one storey) on a well-insulated slab. The slab is raised four or five feet above grade to allow for a tall solar air heater on the south facade of the building. The air heater operates entirely by natural convection, charging a thermal mass situated above it. Since the only exchange of air between the air heater and the rest of the house occurs through the top of the air heater, all air exchange stops when the air heater becomes cold. The thermal mass is located in an overhead attic heat store, about four feet in vertical extent, and covering the whole of the habitable space below. The remainder of the house is assumed to have no thermal mass. The thermal mass in the attic heat store consists of a one foot (0.3 m) thick layer of 1-1/2" to 2-1/2" stones (load: 100 lbf/ft², 4800 n/m²) supported on wire mesh two feet above the floor of the attic heat store. There is a one foot space between the top of the stones and the ceiling of the attic heat store. A thermostatically controlled ducted fan draws hot air from the top of the attic heat store into the living space below. A *flow organiser* permits the coolest air in the attic heat store to pass through the flow of hot air rising from the air heater so that the cool air may fall through the air heater against the glazing. For details of the thermal scheme of the house see <http://geocities.com/davidmdelaney/thermal-cs/thermal-crawl-space-1.html>. For details of the flow organiser see <http://geocities.com/davidmdelaney/flow-organiser/flow-organiser.html>.

Units:

$$\begin{array}{l}
 F \equiv \frac{5}{9} K \quad \text{Btu} \equiv 1054 \text{ J} \quad C \equiv K \quad \text{MMBtu} \equiv 10^6 \text{ Btu} \\
 \\
 \text{kJ} \equiv 1000 \text{ J} \quad \text{kW} \equiv 1000 \text{ W} \quad \text{kWh} \equiv 3.6 \cdot 10^6 \text{ J} \quad \text{GJ} \equiv 10^9 \text{ J} \quad \text{kBtu} \equiv 1000 \text{ Btu} \\
 \\
 R \equiv 1 \frac{\text{ft}^2 \cdot \text{hr} \cdot \text{F}}{\text{Btu}} \quad \text{RSI} \equiv 1 \cdot \frac{\text{m}^2 \cdot \text{K}}{\text{W}} \quad \text{RSI} = 5.673 \cdot R \quad R = 0.1763 \cdot \text{RSI}
 \end{array}$$

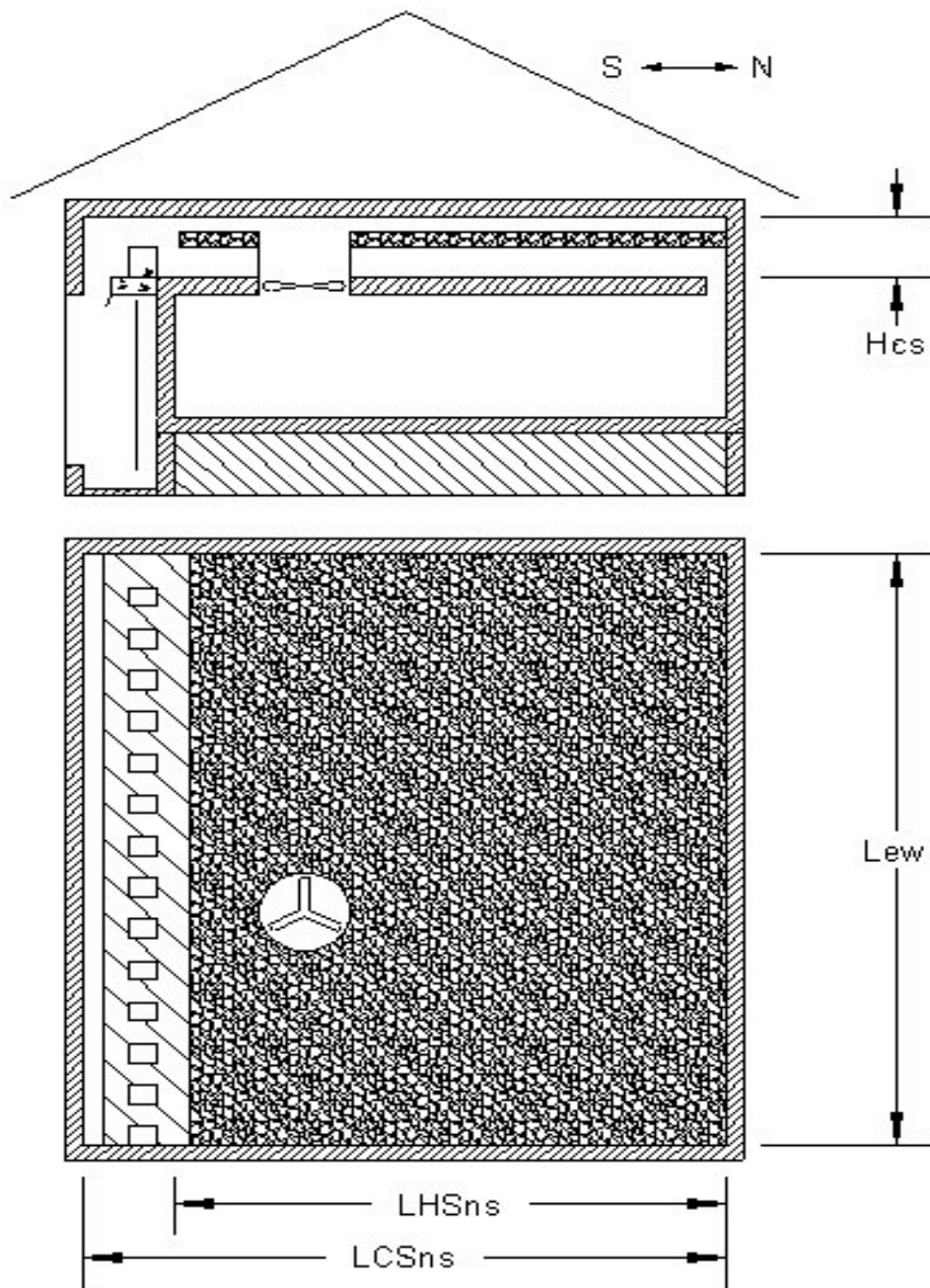
Temperatures:

$T_h := 70 \text{ F}$	$T_h - 32 \cdot \text{F} = 21.1 \cdot \text{C}$	Temperature of the habitable space of the house
$T_{ahs} := 100 \text{ F}$	$T_{ahs} - 32 \text{ F} = 37.8 \cdot \text{C}$	Average temperature of attic heat store
$T_a := 14 \text{ F}$	$T_a - 32 \text{ F} = -10 \cdot \text{C}$	Ambient (outdoor) air temperature. Used only to model temperature decline when there is no sun..
$T_g \approx 50 \text{ F}$	$T_g - 32 \text{ F} = 10 \cdot \text{C}$	Temperature of the ground below the house.
$T_{base} \approx 64.4 \text{ F}$	$T_{base} - 32 \text{ F} = 18 \cdot \text{C}$	Temperature wrt which degree days are measured

Although MathCAD converts automatically between dimensionally consistent units known to it, it cannot do unit conversions that involve addition. So although it converts very well between Fahrenheit and Celsius temperature *differences*, it cannot convert between temperatures. I have chosen to make Fahrenheit the scale in which *temperatures* are entered. I enter *temperature difference data* in whatever scale I obtained the data. I enter degree-days data, for example, in Celsius.

Miscellaneous parameters:

NumberOfPersons ≈ 3	In continuous residence
baseNonSolarHeatGain := 200 W	Human body heat.
electricityUsageHeatGain := 400 W	Electric lights, toaster, microwave, computer, TV, radio, CD player, DVD player, VHS, electric stovetop and oven
nonSolarHeatGain := baseNonSolarHeatGain + electricityUsageHeatGain	
	nonSolarHeatGain = 600 W
thermalCapacity := 22000 $\frac{\text{Btu}}{\text{F}}$	Thermal capacity of the the attic heat store. 22,000 Btu/F is the thermal capacity of 11 tons water, or 10 tonnes water, or 55 tons of stone, or 50 tonnes of stone
	thermalCapacity = 11.6 $\frac{\text{kWh}}{\text{C}}$



Dimensions:

$L_{ew} \equiv 40 \cdot \text{ft}$	$L_{ew} = 12.2 \text{ m}$	The east-west length of the interior of the house
$L_{HSns} \equiv 30 \cdot \text{ft}$	$L_{HSns} = 9.1 \text{ m}$	The north south length of the habitable space
$L_{AHSns} \equiv 35 \cdot \text{ft}$	$L_{AHSns} = 10.7 \text{ m}$	The north south length of the attic heat store
$H_{ahs} \equiv 4 \cdot \text{ft}$	$H_{ahs} = 1.2 \text{ m}$	The height of the attic heat store
$H_{hs} \equiv 8 \cdot \text{ft}$	$H_{hs} = 2.4 \text{ m}$	The height of the habitable space

R-values:

$R_r \equiv 100 \cdot R$	$R_r = 17.6 \cdot \text{RSI}$	Roof
$R_{walls} \equiv 50 \cdot R$	$R_{walls} = 8.8 \cdot \text{RSI}$	Walls of the habitable space
$R_f \equiv 20 \cdot R$	$R_f = 3.5 \cdot \text{RSI}$	Floor of habitable space
$R_{win} \equiv 4 \cdot R$	$R_{win} = 0.7 \cdot \text{RSI}$	Windows
$R_{ahs_wall_inc} \equiv 7 \cdot R$	$R_{ahs_wall_inc} = 1.2 \cdot \text{RSI}$	attic heat store wall increment

Solar collector (air heater) data:

$\text{NumberOfGlazingLayers} \equiv 1$	Not set up to change this.
$R_{ah_glazing} := \text{NumberOfGlazingLayers} \cdot R$	R-value of the air heater glazing
$R_{ah_glazing} = 1 \cdot R$	$R_{ah_glazing} = 0.176 \cdot \text{RSI}$ Not currently used
$\text{TransmissionCoefficient} \equiv 0.85^{\text{NumberOfGlazingLayers}}$	Proportion of incident energy that gets through the glazing.
	$\text{TransmissionCoefficient} = 0.85$
$\text{EnergyRetentionCoefficient} \equiv 0.6$	Guess. Proportion of energy that gets through the glazing that is transferred into the attic heat store.
$\text{airHeaterEfficiency} \equiv \text{TransmissionCoefficient} \cdot \text{EnergyRetentionCoefficient}$	
$\text{airHeaterEfficiency} = 0.51$	Proportion of solar radiation energy that falls on the air heater glazing that winds up in the attic heat store.

Window data:

$$\text{WindowFractionOfFloor} \equiv 0.1$$

$$W_s \equiv \frac{1}{4} \cdot \text{WindowFractionOfFloor} \cdot L_{ew} \cdot L_{HSns} \quad \text{Area of south windows}$$

$$W_s = 30 \text{ ft}^2 \quad W_s = 2.8 \text{ m}^2$$

$$W_{ewn} \equiv \frac{3}{4} \cdot \text{WindowFractionOfFloor} \cdot L_{ew} \cdot L_{HSns} \quad \text{Area of east west and north windows}$$

$$W_{ewn} = 90 \text{ ft}^2 \quad W_{ewn} = 8.4 \text{ m}^2$$

Conductances:

$$\text{East west and north windows:} \quad G_{win_ewn} \equiv \frac{W_{ewn}}{R_{win}} \quad G_{win_ewn} = 22.5 \frac{\text{Btu}}{\text{hr} \cdot \text{F}} \quad G_{win_ewn} = 11.9 \frac{\text{W}}{\text{C}}$$

$$\text{South windows} \quad G_{win_s} \equiv \frac{W_s}{R_{win}} \quad G_{win_s} = 7.5 \frac{\text{Btu}}{\text{hr} \cdot \text{F}} \quad G_{win_s} = 4 \frac{\text{W}}{\text{C}}$$

$$\text{Attic heat store} \quad G_{cs} \equiv \frac{(2 \cdot L_{ew} + 2 \cdot L_{AHSns}) \cdot H_{ahs}}{R_{walls} + R_{ahs_wall_inc}} + \frac{L_{ew} \cdot L_{AHSns}}{R_r}$$

$$G_{cs} = 24.5 \frac{\text{Btu}}{\text{hr} \cdot \text{F}} \quad G_{cs} = 12.9 \frac{\text{W}}{\text{C}}$$

We treat the floor of the attic heat store as if its conductance is 0, since the sum of all energy flows to the habitable space from the attic heat store is exactly equal to the heat losses to the exterior from the habitable space.

$$\text{East west and north walls:} \quad G_{ewn_walls} \equiv \frac{(L_{ew} + 2 \cdot L_{HSns}) \cdot H_{hs} - W_{ewn}}{R_{walls}}$$

$$\text{South wall} \quad G_{s_wall} \equiv \frac{L_{ew} \cdot H_{hs} - W_s}{R_{walls}}$$

$$\text{Floor} \quad G_{floor} \equiv \frac{L_{ew} \cdot L_{HSns}}{R_f}$$

Ventilation load:

Use a well known crude approximation for the enthalpy change of air: "To raise the temperature of one cubic foot of air by one degree Fahrenheit in one minute requires a heating power of one Btu per hour".

$$\text{EnthalpyChangeApprox} \equiv \frac{1 \text{ Btu}}{60 \text{ ft}^3 \cdot \text{F}}$$

Use ASHRAE 62 outdoor air requirement

$$\text{ASHRAE_62_requirement_per_person} \equiv 15 \frac{\text{ft}^3}{\text{min}}$$

$$\text{NewAirVolumeRate} := \text{NumberOfPersons} \cdot \text{ASHRAE_62_requirement_per_person}$$

$$\text{NewAirLossPerDegree} := \text{EnthalpyChangeApprox} \cdot \text{NewAirVolumeRate}$$

$$\text{NewAirLossPerDegree} = 45 \circ \frac{\text{Btu}}{\text{hr} \cdot \text{F}}$$

$$\text{NewAirLossPerDegree} = 23.7 \circ \frac{\text{W}}{\text{C}}$$

Degree days analysis:

"EL" = "energy loss"

$$EL_air(days, ddays) := \text{NewAirLossPerDegree} \cdot ((Th - Tbase) \cdot days + ddays) \cdot day$$

$$EL_air(31, 824 \text{ C}) = 1.9 \cdot 10^9 \text{ J}$$

$$EL_ewn(days, ddays) := \left(\frac{\text{Gewn_walls} \dots}{+ \text{Gwin_ewn}} \right) \cdot ((Th - Tbase) \cdot days + ddays) \cdot day$$

$$EL_ewn(31, 824 \text{ C}) = 1.5 \cdot 10^9 \text{ J}$$

$$EL_s(days, ddays) := \left(\frac{\text{Gs_wall} \dots}{+ \text{Gwin_s}} \right) \cdot ((Th - Tbase) \cdot days + ddays) \cdot day \cdot \frac{3}{4}$$

South wall only loses energy for 3/4 day $EL_s(31, 824 \text{ C}) = 4.2 \cdot 10^8 \text{ J}$

$$EL_tcs(days, ddays) := \text{Gcs} \cdot ((Tahs - Tbase) \cdot days + ddays) \cdot day$$

$$EL_tcs(31, 824 \text{ C}) = 1.6 \cdot 10^9 \text{ J}$$

$$EL_floor(days, ddays) := \text{Gfloor} \cdot (Th - Tg) \cdot days \cdot day$$

$$EL_floor(31, 824 \text{ C}) = 9.4 \cdot 10^8 \text{ J}$$

$$EL(days, ddays) := \left[\begin{array}{l} EL_air(days, ddays) \dots \\ + EL_ewn(days, ddays) \dots \\ + \frac{3}{4} \cdot EL_s(days, ddays) \dots \\ + EL_tcs(days, ddays) \dots \\ + EL_floor(days, ddays) \dots \\ + (-\text{nonSolarHeatGain} \cdot 24 \text{ hr} \cdot days) \end{array} \right]$$

No loss through south wall
for 1/4 of each day

Months := ("Oct" "Nov" "Dec" "Jan" "Feb" "Mar" "Apr")

mnthDays := (31 30 31 31 31 31 30)^T

Solar radiation data (HDdays, irradiancePerDay) from NASA meteorology site,
<http://eosweb.larc.nasa.gov/sse/>, for location 45.316N, 75.666W, (Ottawa, Ontario).

Heating degree days: HDdaysOttawa = (327 536 824 870 751 619 341)^T C

Parameters for tilted

Parameters for tilted solar panels: Radiation on equator-pointed tilted surfaces / RETScreen method, Tilt 90 row:

$$\text{irradiancePerDayOttawa} := (2.38 \quad 1.87 \quad 2.16 \quad 2.96 \quad 3.54 \quad 3.41 \quad 2.85) \frac{\text{kWh}}{\text{m}^2}$$

$$\text{irradiancePerDayOttawa}_2 = 2.2 \frac{\text{kWh}}{\text{m}^2}$$

$$\text{ELm}(\text{mnth}) := \text{EL}(\text{mnthDays}_{\text{mnth}}, \text{HDdaysOttawa}_{\text{mnth}})$$

Oct	ELm(0) = 2•MMBtu	ELm(0) = 2.1•GJ	ELm(0) = 584•kWh
Nov	ELm(1) = 3•MMBtu	ELm(1) = 3.2•GJ	ELm(1) = 881•kWh
Dec	ELm(2) = 4.4•MMBtu	ELm(2) = 4.7•GJ	ELm(2) = 1.3•10 ³ •kWh
Jan	ELm(3) = 4.7•MMBtu	ELm(3) = 4.9•GJ	ELm(3) = 1.4•10 ³ •kWh
Feb	ELm(4) = 4.1•MMBtu	ELm(4) = 4.3•GJ	ELm(4) = 1.19•10 ³ •kWh
Mar	ELm(5) = 3.4•MMBtu	ELm(5) = 3.6•GJ	ELm(5) = 1•10 ³ •kWh
Apr	ELm(6) = 2.1•MMBtu	ELm(6) = 2.2•GJ	ELm(6) = 600.7•kWh

Required size of vertical solar aperture:

$$\text{airHeaterEfficiency} = 0.51$$

$$\text{SolarArea}(\text{mnth}) := \frac{\text{ELm}(\text{mnth})}{\text{airHeaterEfficiency} \cdot \text{mnthDays}_{\text{mnth}} \cdot \text{irradiancePerDayOttawa}_{\text{mnth}}}$$

Oct	SolarArea(0) = 167.1•ft ²	SolarArea(0) = 15.5•m ²
Nov	SolarArea(1) = 331.5•ft ²	SolarArea(1) = 30.8 m ²
Dec	SolarArea(2) = 409.5•ft ²	SolarArea(2) = 38 m ²
Jan	SolarArea(3) = 314•ft ²	SolarArea(3) = 29.2 m ²
Feb	SolarArea(4) = 229.6•ft ²	SolarArea(4) = 21.3 m ²
Mar	SolarArea(5) = 200.5•ft ²	SolarArea(5) = 18.6 m ²
Apr	SolarArea(6) = 148.3•ft ²	SolarArea(6) = 13.8 m ²

Temperature decline with no sun:

Calculate the temperature of the attic heat store assuming only a small amount of miscellaneous non-solar power input.

$$\text{clock}(t) := t - 24 \cdot \text{floor}\left(\frac{t}{24}\right) \quad t = \text{hours elapsed with no sun starting at some midnight}$$

$\text{TH}(\text{Tahs}) := \text{if}(\text{Th} < \text{Tahs}, \text{Th}, \text{Tahs})$ As the temperature of the attic heat store declines, it eventually hits Th , the desired temperature of the habitable space. The temperature of the habitable space then declines in step with the temperature of the attic heat store, remaining equal to it for the remainder of the decline of the temperature of the attic heat store.

$$\text{PwrLossWalls}(\text{Tahs}, t) := (\text{TH}(\text{Tahs}) - \text{Ta}) \cdot \left[(\text{Gewn_walls} + \text{Gwin_ewn}) \dots \right. \\ \left. + ((\text{clock}(t) \geq 9) \cdot (\text{clock}(t) < 15)) \cdot (\text{Gs_wall} + \text{Gwin_s}) \right]$$

South wall loses energy only between 9:00 AM and 3:00 PM

$$\text{PwrLossTCS}(\text{Tahs}) := \text{Gcs} \cdot (\text{Tahs} - \text{Ta})$$

$$\text{PwrLossFloor}(\text{Tahs}) := \text{Gfloor} \cdot (\text{TH}(\text{Tahs}) - \text{Tg})$$

$$\text{PwrAirLoss}(\text{Tahs}) := (\text{TH}(\text{Tahs}) - \text{Ta}) \cdot \text{NewAirLossPerDegree}$$

$$\text{PwrLoss}(\text{Tahs}, t) := \text{PwrLossWalls}(\text{Tahs}, t) \dots \\ + \text{PwrLossTCS}(\text{Tahs}) \dots \\ + \text{PwrLossFloor}(\text{Tahs}) \dots \\ + \text{PwrAirLoss}(\text{Tahs}) - \text{nonSolarHeatGain}$$

$$\text{hours} := 1 \dots 3000$$

$$\text{TCS}_0 := \text{Tahs} \quad \text{Starting temperature for the attic heat store}$$

$$\text{TCS}_0 = 100 \cdot \text{F} \quad \text{TCS}_0 - 32 \text{ F} = 37.8 \cdot \text{C}$$

Difference equation for the temperature of the attic heat store:

$$TCS_{\text{hours}} := TCS_{\text{hours}-1} - \frac{1}{\text{thermalCapacity}} \cdot \text{PwrLoss}(TCS_{\text{hours}-1}, \text{hours} - 1) \cdot 1 \text{ hr}$$

$Thh_0 := Th$ Starting temperature for the habitable space

$$Thh_0 = 38.9 \text{ } ^\circ\text{C}$$

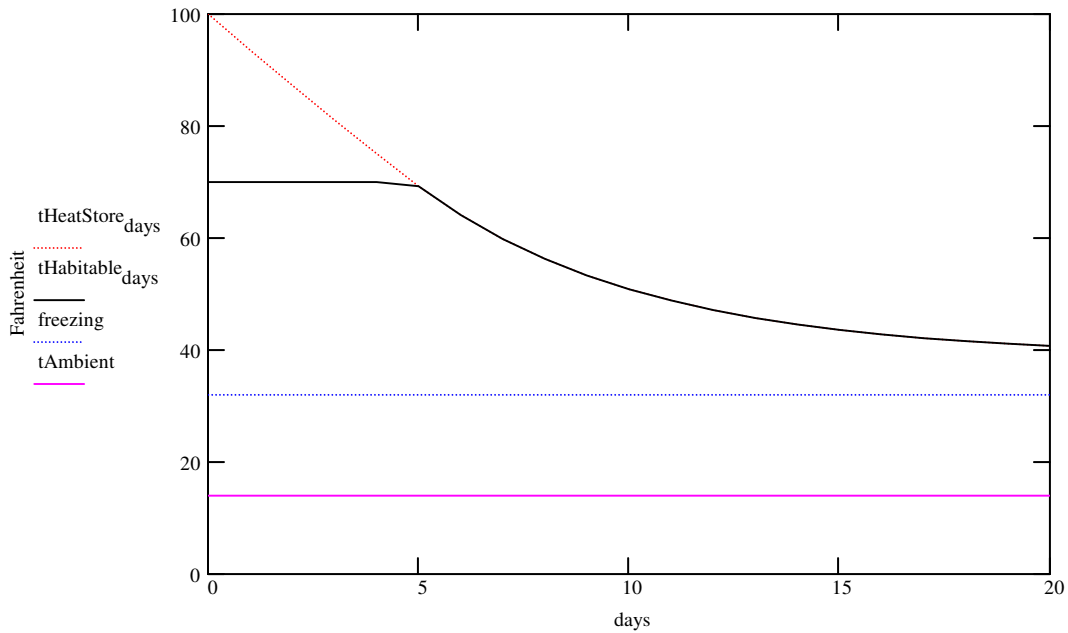
$$Thh_{\text{hours}} := TH(TCS_{\text{hours}})$$

$$t_{\text{HeatStore}} := \frac{TCS}{F} \quad t_{\text{Habitable}} := \frac{Thh}{F} \quad t_{\text{Ambient}} := \frac{Ta}{F} \quad \text{freezing} := 32$$

We have calculated with hour ticks for accuracy, but we graph with day ticks.

$$\text{days} := 0..125$$

$$t_{\text{HeatStore}}_{\text{days}} := t_{\text{HeatStore}}_{\text{days} \cdot 24} \quad t_{\text{Habitable}}_{\text{days}} := t_{\text{Habitable}}_{\text{days} \cdot 24}$$



Temperature (F) decline with no sun

$$t_{\text{Ambient}} = 14 \quad \text{nonSolarHeatGain} = 600 \text{ W}$$

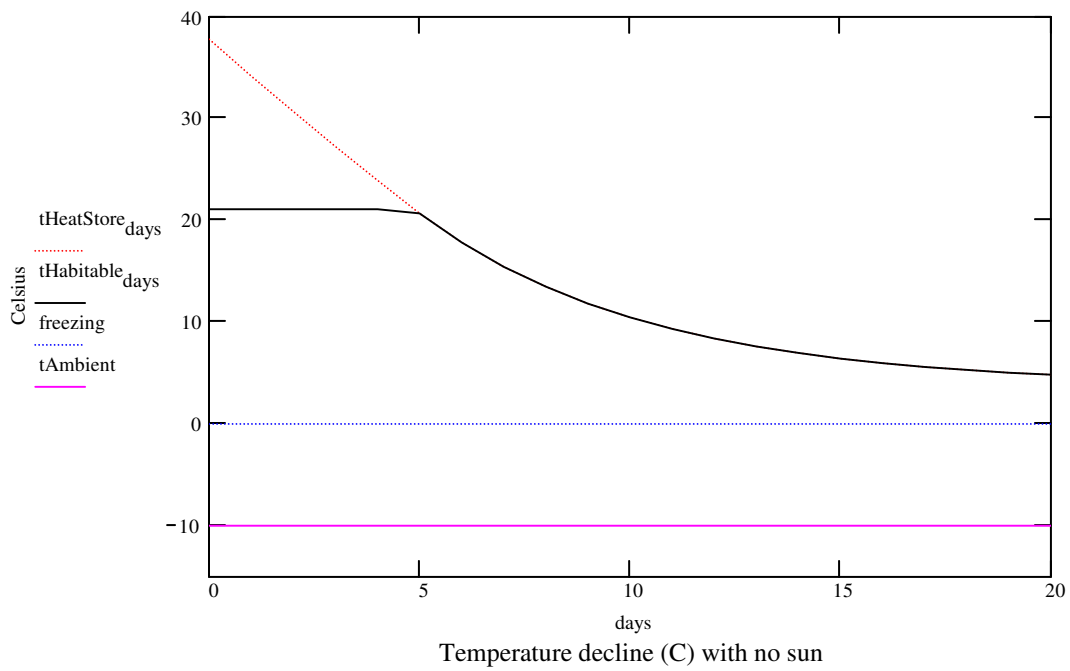
$$\text{thermalCapacity} = 2.2 \cdot 10^4 \frac{\text{Btu}}{\text{F}}$$

$$t_{\text{HeatStore}}_0 = 100 \quad t_{\text{Habitable}}_0 = 70 \quad t_{\text{Habitable}}_7 = 59.8 \quad t_{\text{Habitable}}_{10} = 50.8$$

$$t_{\text{Habitable}}_{31} = 39.1$$

$$t_{\text{HeatStore}} := \frac{T_{\text{CS}} - 32 \text{ F}}{C} \quad t_{\text{Ambient}} := \frac{T_{\text{a}} - 32 \text{ F}}{C} \quad t_{\text{Habitable}} := \frac{T_{\text{hh}} - 32 \text{ F}}{C} \quad \text{freezing} := 0$$

$$t_{\text{HeatStore}}_{\text{days}} := t_{\text{HeatStore}}_{\text{days} \cdot 24} \quad t_{\text{Habitable}}_{\text{days}} := t_{\text{Habitable}}_{\text{days} \cdot 24}$$



$$t_{\text{Ambient}} = -10 \quad \text{nonSolarHeatGain} = 600 \text{ W}$$

$$t_{\text{HeatStore}}_0 = 37.8 \quad t_{\text{Habitable}}_0 = 21.1 \quad t_{\text{Habitable}}_7 = 15.5 \quad t_{\text{Habitable}}_{31} = 3.9$$

Simulation of December with fixed solar aperture and grid on:

$$\text{airHeaterGlazingArea} := 40 \text{ m}^2$$

$$\text{airHeaterGlazingArea} = 430.6 \text{ ft}^2$$

$$\text{solarPwrDensity}_0 := 0 \frac{\text{W}}{\text{m}^2}$$

Between midnight and 1 AM on december 1

$$\text{solarPwrDensity}_{\text{hours}} := \text{if} \left[(\text{clock}(\text{hours}) \geq 9) \cdot (\text{clock}(\text{hours}) < 15), \frac{1}{6 \cdot \text{hr}} \cdot \text{irradiancePerDayOttawa}_2, 0 \frac{\text{W}}{\text{m}^2} \right]$$

$$\text{solarPowerGain}_{\text{hours}} := \text{airHeaterGlazingArea} \cdot \text{airHeaterEfficiency} \cdot \text{solarPwrDensity}_{\text{hours}}$$

$$\text{TAHS}_0 := \text{Tahs}$$

$$\text{TAHS}_0 = 100 \text{ }^\circ\text{F}$$

$$\text{TAHS}_{\text{hours}} := \text{TAHS}_{\text{hours}-1} \dots$$

$$+ - \frac{1}{\text{thermalCapacity}} \cdot \text{PwrLoss}(\text{TAHS}_{\text{hours}-1}, \text{hours}-1) \cdot 1 \text{ hr} \dots$$

$$+ \frac{1}{\text{thermalCapacity}} \cdot \text{solarPowerGain}_{\text{hours}-1} \cdot 1 \text{ hr}$$

$$\text{Thh}_0 := \text{Th}$$

Starting temperature for the habitable space

$$\text{Thh}_0 = 70 \text{ }^\circ\text{F}$$

$$\text{Thh}_0 = 38.9 \text{ }^\circ\text{C}$$

$$\text{Thh}_{\text{hours}} := \text{TH}(\text{TAHS}_{\text{hours}})$$

$$\text{tHeatStore} := \frac{\text{TAHS}}{\text{F}}$$

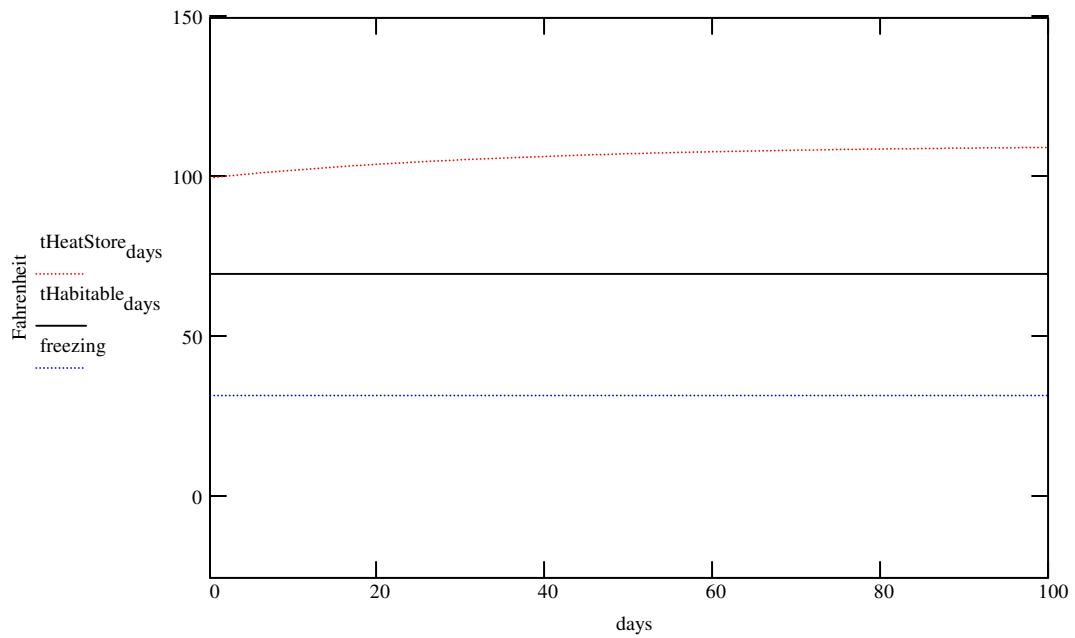
$$\text{tHabitable} := \frac{\text{Thh}}{\text{F}}$$

$$\text{tAmbient} := \frac{\text{Ta}}{\text{F}}$$

$$\text{freezing} := 32$$

$$\text{tHeatStore}_{\text{days}} := \text{tHeatStore}_{\text{days}-24}$$

$$\text{tHabitable}_{\text{days}} := \text{tHabitable}_{\text{days}-24}$$



December with typical (NASA) sun

nonSolarHeatGain = 600 W

$t_{HeatStore_0} = 100$

$t_{Ambient} = 14$

$t_{Habitable_0} = 70$

$t_{Habitable_{10}} = 70$

$t_{Habitable_{31}} = 70$

airHeaterGlazingArea = 40 m^2

$t_{HeatStore_{124}} = 109.8$

